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To: Professor J. E. Broadwell
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Subject: Experimental Determination and Investigation of a Sinusoidally Varying Gust, Report of

1. This report is submitted in partial fulfillment of the requirements of the United States Naval Postgraduate School for the degree of Master of Science.

2. These investigations proved the feasibility of generating a sinusoidal gust.

Respectfully submitted,

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EXPERIMENTAL GENERATION AND INVESTIGATION OF A SINUSOIDALLY VARYING GUST

by

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TABLE OF CONTENTS

EXPERIMENTAL GENERATION AND INVESTIGATION OF A SINUSOIDALLY VARYING GUST

SUMMARY

The objective of this project was to generate and study sinusoidal gusts in the wind tunnel. The sinusoidally varying gusts were produced by means of a row of harmonically oscillating vanes. The basic flow parameters were determined and wave properties analyzed. From the test data it was possible to define a test volume in the flow, where the wave properties were reasonably constant and a response model might be placed. The amplitude recovery through this test section was a function of the reduced frequency, k; and for a k = 0.20 was determined to be 1/3 of the input amplitude. This is considered to be adequate.

This study was conducted in the low-speed, low-turbulence tunnel of the University of Michigan during the spring of 1958. The investigations were made under the supervision of Professor Julius D. Schetzer of the Department of Aeronautical Engineering.

EXPERIMENTAL GENERATION AND INVESTIGATION OF A SINUSOIDALLY VARYING GUST

INTRODUCTION

Upon the suggestion of Professor Julius D. Schetzer, of the Department of Aeronautical Engineering, University of Michigan, the authors undertook the construction of a gust generator and an investigation of the flow field produced by it.

The primary purpose of the work was to determine the feasibility of generating a sinusoidally varying gust of reasonable magnitude. Information was also desired to determine a desirable test section where a model could be installed in order to measure the response to a sinusoidal gust input.

Developments in the theory of generalized harmonic analysis appear adaptable for extending the analysis of gust effects beyond the discrete-gust case to the case of continuous turbulence. The sharp-edge discrete-gust theory is not too realistic a consideration since atmospheric turbulence is composed of continuous random motion rather than a series of single gusts. Therefore emphasis recently has been placed on generating oscillating flows, and in particular a sinusoidally varying flow. Previous to this time a great deal of work has been devoted to "maximum" gust velocities for airworthiness requirements (Refs. 1 and 2), and to the response of an aircraft to single gusts (Ref. 3). A vehicle traveling through the atmosphere experiences a load history which is a randomly oscillating function of time. More realistic results would be obtained then from a study of the response to a motion of a sinusoidal type rather than

from that of a single gust.

One of the most recent studies in this field was conducted at the Massachusetts Institute of Technology by Hakkinen and Richardson (Ref. 4), in which measurements were made of a sinusoidally oscillating downwash and the lift produced on a simple rigid airfoil. Flow measurements were made directly downstream from a sinusoidally oscillating vertical 1-foot-chord, 5-foot-span wing. This experiment failed to show conclusive results as the wake effects and background turbulence of the tunnel were too high to provide good measurement of the desired quantities. It was found that the velocity and lift sensing units could not satisfactorily distinguish these quantities from the relatively high free-stream turbulence of the tunnel.

It was believed by the authors, that more reliable data of the sinusoidal flow could be observed and obtained if tests were conducted in a relatively low turbulence tunnel in an area of the flow which was not directly behind the trailing edge of the generating airfoils where the turbulence level was necessarily high. In order to avoid this high turbulence condition it was decided to measure the characteristics of the sinusoidal cross velocity fluctuations in a low-turbulence wind tunnel at a station midway between two oscillating airfoils. From the measurements obtained it was possible to obtain experimental values of the phase angle lag and attenuation of the flow.

The work was conducted in the low-speed, low-turbulence tunnel of the University of Michigan Aerodynamics Research Laboratory, Ann Arbor, Michigan during January through April 1958. It was performed as part of the third year curriculum in Aeronautical Engineering of the United States Naval Postgraduate School, Monterey, California, as partial fulfillment of the requirements for a Master of Science degree. It was financed by the Bureau of Aeronautics, Navy

Department, Washington, D. C.

We would like to express our appreciation to Professor J. D. Schetzer for his general overall guidance and enthusiastic interest in the project. Professors J. E. Broadwell and M. S. Uberoi, L. C. Garby, and S. B. Wallis also contributed many helpful suggestions. We wish to thank the Aeronautical Engineering Machine Shop staff for their generous assistance in the construction of the gust generator.

TABLE OF SYMBOLS

c	chord, ft.
C_i	cosine integral, $C_i \xi = -\int_{\xi}^{\infty} \frac{\cos \xi'}{\xi'} d\xi'$
C_L	Lift coefficient
$H_0^{(2)}$, $H_1^{(2)}$	Hankel functions
k	reduced frequency, $k = \frac{\omega}{U}(c/2)$
l	streamwise distance (from midchord), ft.
q	dynamic pressure, psi.
Re	Reynolds number
Si	sine integral, $Si(\xi) = \int_0^{\xi} \frac{\sin \xi'}{\xi'} d\xi'$
t	time, sec.
U	stream velocity, ft/sec.
U_w	propagation wave velocity, ft/sec.
u	streamwise velocity fluctuation, ft/sec.
w	crosswise velocity fluctuation, ft/sec.
x	streamwise coordinate, ft.
y	spanwise coordinate, ft.
z	normal coordinate, ft.
z'	crosswise distance measured from vane, ft. positive sense, inboard.
ξ'	streamwise distance variable,
ϕ	phase angle, degrees
ω	circular frequency, radians/sec.
$\lvert\theta_i\rvert$	magnitude of input oscillation

TABLE OF SYMBOLS (Continued)

$\lvert \theta_o \rvert$	magnitude of output gust angle
$\left\lvert \frac{\theta_o}{\theta_i} \right\rvert$	amplitude ratio
$\bar{\Omega}$	reduced circulation amplitude, ft/sec.
λ	wavelength, ft.

SUBSCRIPTS

v	behind the vane
a	on the airfoil

GENERAL EXPERIMENTAL EQUIPMENT

The general layout of equipment and apparatus is shown in Fig. 1.

Wind Tunnel

All measurements were made in the 24-by-24-inch low-speed low turbulence tunnel of the University of Michigan Aerodynamics Research Laboratory. This open-return tunnel has a maximum speed of approximately 90 feet per second. The turbulence level is 0.03% at a tunnel velocity of 50 feet per second. The tunnel is driven by a 5 horsepower motor-driven propeller and the velocity is controlled in the test section by controlling the speed of the fixed pitch propeller by means of a Reeves Varispeed drive. Descriptions of the mechanical and instrumentative details of the tunnel are given in Refs. 5, 6, and 7.

The tunnel velocity was measured by means of a pitot-static probe located immediately adjacent to the hot-wire probe.

The gust generator built for this project was placed at the midsection of the tunnel by opening the tunnel at this point, rolling the mobile generator stand in and resealing the tunnel. A sketch of this tunnel circuit is shown in Fig. 2.

To designate positions in the test section, the following rectangular coordinate system is used, see Fig. 3. The origin of the system is located on the centerline of the tunnel, at the midchord position of the flipper vanes. The scale units along all axes are in feet. The coordinate axes are designated as follows:

x axis - in the direction of the flow, positive downstream.

y axis - in the vertical direction, positive downward.

z axis - in the horizontal plane and normal to the flow, positive sense from the right hand rule.

GUST GENERATOR

The basic component of the harmonic gust generator was a set of six vertical, symmetrical airfoils, hinged about their quarter chord line. The vanes were spaced 4-1/4 inches apart and driven through a mechanical linkage by means of a 1/8 horsepower constant speed motor. Various vane oscillation frequencies were available through a set of pulley cones; however, an upper limit of approximately 600 revolutions per minute was set by the vibrational modes set up in the vanes at this relatively high oscillation rate.

After a few preliminary runs, vanes #3 and #4 were removed, leaving two outboard vanes on either side of the tunnel centerline. (See Fig. 3). This was done in order to provide a reasonably large, relatively low turbulence area in which to make test measurements. Velocity fluctuations were introduced into the mean air flow by means of the vertical vanes of the gust generator. The linkage mechanism and drive components used to develop this motion are shown in Fig. 4.

Auxiliary Equipment

A KayLab Model 111 DC Decade Amplifier was used for the amplification of the hot-wire signals. This type amplifier was specifically designed to accept a small DC voltage from a electro-mechanical source, and amplify this signal. This provided a voltage output which is used to drive recording equipment. A large amount of feedback provides a very precise and stable gain. Frequency response of the amplifier was flat from 1 to 40 kilocycles for a gain in the range of 300 to 1000. This amplifier is shown together with some of the other circuit components in Fig. 1 (Item A).

The hot-wire apparatus used for flow measurements was similar to that developed and built by the National Bureau of Standards in 1940, with the exception that the AC amplifier stage was not used and the DC model described above was substituted. This was necessary because of the low range of frequencies that were to be used in these tests (0 to 50 cycles per second). The portable flow measuring apparatus is shown in Fig. 1 (Item B).

Fig. 5 shows two sample recordings of the vane input motion and the hot wire-signal taken simultaneously by means of a Sanborn Twin-Viso Recorder (Model 60). This is a two channel, rectangular coordinate, variable speed, direct writing recorder. The recorder has an internal three stage DC push pull amplifier. Push-pull or single sided inputs may be amplified and through proper use of the attenuating network, signals up to 250 volts may be accommodated without damage to the recorder. The Sanborn recorder is shown in Fig. 1 (Item C).

Measurement of Velocity Fluctuations

The flow velocity fluctuations were measured by means of a directionally sensitive hot-wire probe. The probe was constructed from four piano wires forced into a ceramic rod. To the upstream tips of the probe were soldered two platinum Wollaston wires, approximately 0.10 inch-long and .0003 inch in diameter, in the form of an (X). Every effort was made to insure the wires were of approximately the same resistance. The heating current for these wires was furnished by means of a 24-volt aircraft battery, one battery being used for each wire. This arrangement yielded a reasonably stable power source. The probe was mounted on the bottom of a vertical support which was mechanically

linked to a large protractor scale on the upper surface of the tunnel. The probe could then be swung through a known calibration arc for each run. The hot-wire probe could be moved both cross-stream (z direction) and downstream (x direction) thus permitting a complete survey of the flow fluctuations behind the moving vanes. Fig. 6 shows the hot wire probe and protractor scale centered in position in the flow channel.

Facilities were available in the Aerodynamics Laboratory for the manufacture of the hot-wire probes. After several abortive attempts, adequate proficiency was attained in the making of the hot-wire probes and proper precautions were observed to insure reasonably long life for the fragile instruments.

Measurement of Vane Variations

It was desired to record simultaneous traces of the flipper vane motion and the flow at a particular downstream position. The trace of the vane motion was recorded on the upper trace of the two channel Sanborn Recorder. The signal was provided by means of a precision potentiometer attached to the upper end of the #2 vane and powered by a 24 volt battery source.

Measurement of Phase Angle Variations

To obtain measurements of the sinusoidal flow produced by the oscillations of the cascade of vanes, a hot-wire probe was installed downstream of the vane section. The probe was located at a midstream position relatively free of wake effects, thereby avoiding the high turbulence area directly behind the vanes.

The change of flow direction in the plane of the inclined wires is measured as the difference in potential between the two wires. The potential difference is then fed to the amplifier and then to the Sanborn recorder, where the flow pattern

was recorded on the lower channel trace. A simultaneous trace of the vane input motion was recorded on the upper channel trace, as previously mentioned. It was then possible to obtain phase angle and amplitude ratio data very simply and directly.

PROCEDURE

For measuring the sinusoidal flow variation and the associated phase angle lag and amplitude at specified stations along the x axis in the flow, the equipment described in the preceding sections was used. Prior to commencing test runs, a cross channel survey was made with the mobile gust generator operating to determine the thickness of the wall boundary layer at flow speeds of 26 and 42 feet per second. The boundary layer was determined to be approximately 4 inches thick on the test section walls, this provided an adequate undisturbed flow channel.

The hot-wire probe was oriented on the centerline of the flow channel test section and its streamwise position varied from $l/c=2.26$ to $l/c=15.1$ chordlengths in one chordlength increments. The tests were made at tunnel flow velocities which ranged from 37.3 feet per second to 69.0 feet per second. The average Reynolds Number based on the vane chord was 75,000. The oscillations of the flipper vanes which were symmetrical about the centerline were varied through three different amplitude settings, 5.63, 6.70, and 7.63 degrees. The oscillation frequency of the vanes were varied for different runs with speeds of 327, 493, and 584 cycles per minute being used.

At the commencement of each run it was necessary to calibrate the installation by setting the tunnel at the desired test speed and swinging the hot-wire probe through a series of known angles to the flow and recording these angular displacements on the individual traces. This calibration procedure was based on the assumption that the flow was initially parallel to the tunnel centerline. The zero reference position of the hot-wire probe was determined by adjusting the probe until a symmetrical calibration was obtained.

A sinusoidal motion was then imposed on the flow by means of the gust generator and as soon as the tunnel dynamic pressure reading stabilized simultaneous traces were recorded of the vane input motion and the induced flow pattern in the test section.

DISCUSSION AND RESULTS

General

The equation of a sinusoidal, plane wave is:

$$w(\ell,t) = |w| \, \mathcal{R}e \; e^{i(\omega t + \phi)} = |w| \cos(\omega t + \phi) \tag{1}$$

Where w is the amplitude of the oscillation velocity, ω is the angular frequency and ϕ is the phase angle lag which would be observed when moving downstream. Equation (1) is that of a wave propagating downstream. In a wind tunnel the velocity of propagation is not necessarily the flow velocity, and indeed in the present investigation, this propagation velocity was found to be some 18% less than the flow velocity. To completely define the wave it is necessary to specify the wave length, phase propagation velocity, and amplitude.

As shown in Fig. 4 this investigation employed the use of an oscillating cascade of airfoils to produce the sinusoidally varying wave in the wind tunnel. The use of the hot-wire apparatus necessitated the measurement of the flow angle of the wave rather than the wave velocity, w. These measurements were taken at various stations downstream of the cascade while simultaneous measurements of the cascade oscillations were recorded. The cascade oscillations may be described by:

$$\theta_i = |\theta_i| \, e^{i\omega t} \tag{2}$$

Where $|\theta_i|$ is the magnitude of the vane oscillations and ω is the angular frequency of the airfoils. The data obtained from the hot wire apparatus may be expressed as:

$$\theta_o = |\theta_o| \, e^{i(\omega t + \phi)} \tag{3}$$

Where $|\theta_o|$ is the magnitude of the output flow angle and ϕ the angular phase difference between the input oscillations and the measured flow oscillations. In order for Eq. 3 to accurately describe the test data, it is necessary to determine whether the wave, which is recorded at a particular $1/\frac{c}{2}$ station, has harmonic variation and is, in fact, described by $\theta_o(t) = |\theta_o| \, e^{i\omega t}$. A comparison of the wave from the hot wire with that of the cascade trace showed that the waves were similar. Dividing Eq. 2 and Eq. 3 results in the dimensionless quantity $\left|\frac{\theta_o}{\theta_i}\right|$ herein called amplitude ratio.

$$\frac{\theta_o}{\theta_i} = \left|\frac{\theta_o}{\theta_i}\right| e^{i\phi} \tag{4}$$

The time variable has thereby been eliminated as shown by Eq. 4. The desired quantity is the gust velocity, w. For small gust angles, $\theta_o = \frac{w}{U}$. Therefore

$$w(\ell) = |w| e^{i\phi} = |\theta_o| U e^{i\phi} = \left[\left|\frac{\theta_o}{\theta_i}\right| |\theta_i| U\right] e^{i\phi} = \left[\left|\frac{\theta_o}{\theta_i}\right| |\theta_i| U\right] \cos\phi \qquad (5)$$

If algebraic expressions can be found for $\left|\frac{\theta_o}{\theta_i}\right|$ and ϕ, Eq. 5 represents an exact analytical expression for the wave in the wind tunnel. For our purposes, this wave must be a good approximation of a sine wave. This requirement dictates that the quantity $\left[\left|\frac{\theta_o}{\theta_i}\right| |\theta_i| U\right]$ be constant, or nearly so and that ϕ vary in such a manner as to keep the wave length constant with distance downstream. The purpose of this investigation then, is to determine by experiment, whether or not such a region exists in the flow field and if so, to devise a procedure for predicting the form of the wave for changes in the parameters involved.

<u>Theoretical Considerations</u>

Ashley (Ref. 8) has developed theory describing the flow behind a single two dimensional sinusoidally oscillating airfoil. This theory does not adequately describe the present oscillating cascade of airfoils, but some important conclusions may be reached if the theory is considered.

According to Ashley, the downwash in the flow field behind a two dimensional wing oscillating sinusoidally can be expressed as follows, provided that the point of observation is sufficiently far behind the wing:

$$W(\ell,t) = \frac{-\bar{\Omega}}{2\pi} e^{i\omega t} \left\{ e^{-ik}\left[1 - \sqrt{\frac{\frac{2\ell}{c}-1}{\frac{2\ell}{c}+1}}\right] + k e^{-i\left(\frac{2k\ell}{c}\right)}\left[\frac{\pi}{2} + Si\left(\frac{2k\ell}{c}-k\right) - iCi\left(\frac{2k\ell}{c}-k\right)\right]\right\} e^{-\frac{2kz'}{c}} \qquad (6)$$

ω is the angular frequency of the oscillation, l the distance downstream from midchord, c the chord of the airfoil, z' the distance above or below the airfoil, k the reduced frequency ($k = \omega c/2U$), Si and Ci the sine and cosine integrals respectively, and the reduced circulation amplitude is

$$\bar{\Omega} = \frac{4\int_{-1}^{1}\sqrt{\frac{1+\xi'}{1-\xi'}}\, w_a(\xi')\,d\xi'}{i\pi k\left[H_1^{(2)}(k)+iH_0^{(2)}(k)\right]} \tag{7}$$

where $w_o(\xi')$ is the local amplitude of the wing oscillation velocity as a function of the chordwise position $\xi' = x/c/2$, and $H_1^{(2)}$ and $H_0^{(2)}$ are Hankel functions of the first and zero index respectively. In the present case of oscillatory motion about the quarter chord line with maximum angular amplitude $|\theta_i|$

$$w_a(\xi') = (\xi' + \tfrac{1}{2})(\tfrac{c}{2})(i\omega|\theta_i|) \quad \text{ft/sec.} \tag{8}$$

then

$$\bar{\Omega} = \frac{4U|\theta_i|}{H_1^{(2)}(k)+iH_0^{(2)}(k)} \quad \text{ft/sec.} \tag{9}$$

Substituting Eqs. (8) and (9) into (6), the complex downwash becomes:

$$w(l) = -\frac{2}{\pi}\left[\frac{U|\theta_i|e^{-ik}}{H_1^{(2)}(k)+iH_0^{(2)}(k)}\right]\left\{\left[1-\sqrt{\frac{\frac{2l}{c}-1}{\frac{2l}{c}+1}}\right]+ke^{-i\left(\frac{2kl}{c}-k\right)}\left[\frac{\pi}{2}+Si\left(\frac{2kl}{c}-k\right)-iCi\left(\frac{2kl}{c}-k\right)\right]\right\}e^{-\frac{2kz'}{c}} \tag{10}$$

For small angles, $\theta_o = w/U$ and the input, θ_i may be described in terms of the velocity on the wing: $\theta_i = \dfrac{\int_{-1}^{1} w_a(\xi')\,d\xi'}{U} = ik|\theta_i|$

Hence

$$\frac{\theta_o}{\theta_i} = \frac{-2}{\pi k}\left[\frac{e^{-i\left(k+\frac{\pi}{2}\right)}}{H_1^{(2)}(k)+iH_0^{(2)}(k)}\right]\left\{\left[1-\sqrt{\frac{\frac{2l}{c}-1}{\frac{2l}{c}+1}}\right]+ke^{-i\left(\frac{2kl}{c}-k\right)}\left[\frac{\pi}{2}+Si\left(\frac{2kl}{c}-k\right)-iCi\left(\frac{2kl}{c}-k\right)\right]\right\}e^{-\frac{2kz'}{c}} \tag{11}$$

The amplitude ratio $\left|\frac{\theta_o}{\theta_i}\right|$ may be thought of as the fraction of the input amplitude which is recovered. It may be seen that the amplitude ratio may be expressed by

$$\frac{\theta_o}{\theta_i} = \left\{\left[\left|\frac{\theta_o}{\theta_i}\right|\left(\frac{2l}{c}, \frac{2kl}{c}, k\right)\right] e^{i\phi}\right\} e^{-\frac{2kz'}{c}} \tag{12}$$

where the phase angle ϕ , is a function of 2kl/c and k only. The investigation showed that the area of interest was sufficiently far downstream so that the term $1-\sqrt{\frac{\frac{2l}{c}-1}{\frac{2l}{c}+1}}$ is, for all practical purposes equal to zero. Therefore, the basic parameters were taken as 2kl/c and k. 2kl/c is a nondimensional length in the x direction, which is stretched or shortened according to the value of k chosen. The characteristic length is c/2. The term $e^{-2kz/2}$ indicates an exponential decay of the amplitude ratio with distance above or below the two dimensional wing. It would indicate that, for a single oscillating airfoil, the attenuation is more severe for the higher values of reduced frequency. The theoretical calculation of the attenuation of the amplitude ratio behind a cascade of airfoils is beyond the scope of this paper and will not be pursued further except in a qualitative manner.

Eq. 12 also indicates that the phase angle is unaffected by z. Data taken at various values of k and z, show that the phase angle is independent of z. Therefore, it is evident that the theoretical phase angle calculations for the single airfoil should also describe the conditions for the cascade of airfoils.

Eq. 11 is based upon the theoretical lift curve slope of 2π . The theoretical curves plotted in Figs. 7, 8, and 14 were corrected for a lift curve slope of 5.73/rad.

Phase Angle

The previous theoretical discussion indicated the phase angle ϕ , to be a function of 2kl/c and k. For the values of reduced frequencies tested, the phase angles appeared to be a function of 2kl/c only but it is possible that if the range of reduced frequencies tested had been greater, the influence of k might have been demonstrated. Fig. 7 shows a plot of phase angle versus 2kl/c for values of k of 0.0915, 0.12, 0.15, and 0.20. These points all fell on the same straight line on log-log paper from which the equation of the curve was determined to be

$$\phi = 87\left(\frac{2kl}{c} - 0.25\right)^{.848} \quad \text{degrees} \qquad (13)$$

As discussed previously, it is necessary that the variation of ϕ with distance downstream be linear if the wave is to be a sine wave. Subsequent discussion will show that the variation in wave length is so small as to be neglible.

Theory indicates that there should be no phase shift in the z direction. Test runs, at an $\frac{1}{c/2}$ of 13, confirm this. Therefore, the phase angle variation should correspond with the theoretical calculations based on Eq. 11. Fig. 7 shows that the experimental curve gives slightly greater phase angles than would be expected from the theory.

The phase angle data, plotted in Fig. 7 are considered to be quite consistant. It is felt, however, that the scatter could have been reduced if a greater paper speed were available on the Sanborn recorder. A sample of the recorded data is shown in Fig. 5.

Amplitude Ratio

Amplitude ratio data are shown in Figs. 9 through 14. It can be seen that

the amplitude ratio varies markedly with both 2kl/c and k. The data for a k of 0.15 were obtained from three different input amplitudes, and two different angular frequencies. If the wind tunnel walls had a pronounced effect, it would seem reasonable to assume that this effect would be greater for the waves with the greater amplitude. The theory indicates that the amplitude ratio is not a function of the input amplitude and test data confirms this. Thus it would seem, for a given configuration at least, that the wind tunnel wall effect is relatively constant.

Since the theory is based on a single oscillating airfoil, it is apparent that it is invalid for the present case. However, some knowledge may be gathered by comparing the data with this theory. Fig. 8 shows test data for both the center-line and behind one of the inboard vanes. The theoretical curve is also shown. It can be seen that the test curve is considerably higher than theory would predict for a single airfoil. This would indicate that the additional airfoils in the cascade have a pronounced effect and that this effect is to increase the amplitude ratio over what the single airfoil theory predicts. This is not too surprising a result. Since it is desirable to produce as large a flow angle defelection as possible, it would seem that there is definite advantage to having more than two airfoils in the cascade.

Fig. 16 shows the variation of amplitude ratio in the z direction. Useful data could not be obtained directly behind the vanes so the curves were extrapolated in this area with the aid of Fig. 15. The fact that the test data in Fig. 15 fell on one line as a function of 2kz/c, indicates that as theory predicted, the attenuation in the z direction is a function of 2kz/c only. However, it is not that exact function

of 2kz/c given in Eq. 11, but shows considerably greater decay.

Useful wave data were obtained within approximately one chord length either side of the centerline. Outside this area, random fluctuations became more in evidence until, directly aft of the vane, the data were almost unreadable. This suggests that the spacing of the airfoils might be decreased by a chord length and still provide a good uncluttered sine wave output. This would have the advantage of increasing the amplitude ratio at the centerline of the tunnel.

For a k of 0.20, a distance of one chord length either side of the tunnel centerline results in a 3% variation in amplitude ratio.

It was noted that the data were poor for values of k less than 0.10 and greater than 0.20. No explanation for this phenomena can be advanced. A sample of the amplitude trace from the Sanborn recorder, for a value of k of 0.0915, is shown in Fig. 5. It appears that another wave of the same frequency as that of the basic wave, is superimposed. This trace can be contrasted with the faithful reproduction of the input motion shown in the lower portion of Fig. 5 which is for a k of 0.15.

The most useable and consistant amplitude ratio data, then, occured in the range of k from 0.15 to 0.20. Physical limitations on the maximum angular frequencies prohibited the taking of data at values of k greater than 0.22. When attempts to increase the value of the reduced frequency by decreasing the wind tunnel velocity were made, a decided Reynolds Number effect was introduced. This effect was to reduce the slope of the phase angle curve and to lower the amplitude ratio curves. It is thought that the Reynolds Number effect could have been reduced by constructing the airfoils with a rough surface. (Ref. 10)

The requirement that the amplitude ratio be as nearly constant as possible fixes the streamwise test area. Fig. 14 indicates that the center of the test area should probably be at a 2kl/c of 3.25. A typical model placed in this position, would experience an amplitude ratio variation of approximately 1% over its length.

<u>Wave Length and Propagation Velocity</u>

The wave, as computed from Eq. 5 is shown in Fig. 17, in nondimensional form and in Fig. 18 in the physical plane. The equation of a plane wave is

$$w = |w| \cos\left(\frac{2\pi U_w}{\lambda} + \frac{2\pi}{\lambda}\ell\right) \tag{14}$$

where w is the amplitude of the oscillation, U_w is the phase or propagation velocity of the wave, and λ is the wave length. Comparison of Eq. 1 with Eq. 14 gives:

$$\lambda = \frac{2\pi\ell}{\phi/57.3} \tag{15}$$

and

$$U_w = \frac{\omega\ell}{\phi/57.3} = \left(\frac{2\kappa\ell}{c}\right)\left(\frac{U}{\phi/57.3}\right) = \left[\frac{57.3\left(\frac{2\kappa\ell}{c}\right)}{\phi}\right]U \tag{16}$$

Substitution of Eq. 13 into Eqs. 15 and 16 gives:

$$\frac{U_w}{U} = \frac{0.658\left(\frac{2\kappa\ell}{c}\right)}{\left(\frac{2\kappa\ell}{c} - 0.25\right)^{.848}} \tag{17}$$

and

$$\left(\frac{\lambda}{c/2}\right)\kappa = \frac{4.13\left(\frac{2\kappa\ell}{c}\right)}{\left(\frac{2\kappa\ell}{c} - 0.25\right)^{.848}} \tag{18}$$

Eqs. 17 and 18 are plotted in Figs. 19 and 20 respectively. The variation in the phase velocity was 3.7% over the test area. For values of k of 0.15 and 0.20, the wave length at mid point of the test area was 17.6 and 13.2 chord lengths respectively. The variation in wave length over the test area was 3.8%.

It can be concluded that the wave developed in the test section was a very close approximation of a sine wave. This method of wave production can undoubtedly be used to produce a sine wave in which a model may be placed for aircraft response measurements. The limited range of wave lengths is regretable but not too serious, in that a conformal mapping method exists which will transform the results of a wave of one wave length to those of another wave length.

Selection of Parameters for Model Testing

Once the test area has been selected, one need only select a value of reduced frequency which fits his needs. In view of the rather small values of $|\theta_o|$ produced, it seems likely that the selection of the parameters to maximize this quantity might be the primary consideration. With a model in the wind tunnel, θ_o becomes the variation in the angle of attack which the model would experience. Eq. 5 shows that:

$$\alpha = \theta_o = \frac{w(\ell)}{U} = \frac{|\theta_o|}{|\theta_i|}|\theta_i| \cos \phi \qquad (19)$$

where α is the variation in angle of attack of the model. The only practical observed values of reduced frequency lie in the range from 0.15 to 0.20. Fig. 14 indicates that the amplitude ratio varies negligibly with k in this range. The maximum value of $|\theta_i|$ is dependent upon the $C_{L_{max}}$ of the airfoils and the Reynolds Number.

The selection of the amplitude of the α variation, fixes the model lift coefficient amplitude. The amplitude of the total lift, at a given k, can be further varied by varying the wind tunnel velocity. But, this requires varying ω at the same time to keep k constant.

It appears that the choice of k is almost entirely dependent upon the wave length required. A higher value of k results in a shorter wave length. It is postulated that the phase angle data would show little change for other test set-ups, but that considerable variation might appear in the amplitude ratio data for other boundary conditions. If the phase angle were unchanged, this means that the phase propagation velocity and wave lengths would also remain unchanged. Changing the amplitude ratio data would affect the magnitude of the wave only.

The test area was chosen on the basis of minimum variation of amplitude ratio. If there is an objection to this area because of the fact that the phase velocity is less than the tunnel velocity, it is possible that the phase velocity further downstream would approach the free stream velocity (see Fig. 20). Extrapolation of the test data indicates that this would occur at a 2kl/c of about 11.0. Placing a model this far downstream has several disadvantages, however. One is that the amplitude ratio would most certainly be small. A second is that the test data indicated that the quality of the data deteriorates as $l/\frac{c}{2}$ is increased.

There is little choice in the z direction. A model one chord length high would have an approximate amplitude ratio variation of 3% in the z direction. There is only a slight variation in this quantity with lower values of k. Therefore there is no problem in selecting the value of k. The main consideration is to keep the model as close as possible to the centerline of the tunnel.

Accuracy of Measurement

It is difficult to estimate the errors due to the equipment itself, but it seems quite certain that these equipment errors were only a fraction of the total error. The reading error for the phase angle data is estimated to be plus or minus 5 degrees, while that for the amplitude is estimated to be 0.3 degrees. The accuracy of the phase angle data could be improved by increasing the run-out speed of the recording paper on the Sanborn recorder. The maximum paper speed was 100 mm/sec. which resulted in 33 degrees per millimeter for the highest angular frequency used.

CONCLUSIONS AND RECOMMENDATIONS

The equipment and measurement methods used for measuring the sinusoidal flow variation were found to satisfactory. The equipment appeared to be very reliable in reproducing data for similar runs and for rechecking erratic points. It is felt that the propagating wave can be predicted with reasonable accuracy.

The best test area was selected to extend from a 2kl/c of 2.45 to 4.05 in the streamwise direction and plus or minus one chord length in the z direction. The maximum change of any of the wave properties over the test area was 3.8%. Therefore the wave is a very close approximation of a true sine wave and it is felt that this method of wave production would be suitable for the testing of model response to sinusoidal gusts.

The effect of the Reynolds Number parameter could, in all probability, be reduced by constructing the airfoils with a rough surface. The present four airfoil configuration should be retained although the possibility of decreasing the

airfoil spacing should be considered. The decreased spacing would increase the amplitude ratio.

It is felt that a system for continuously varying the airfoil RPM would be superior to the pulley cone method used in this investigation.

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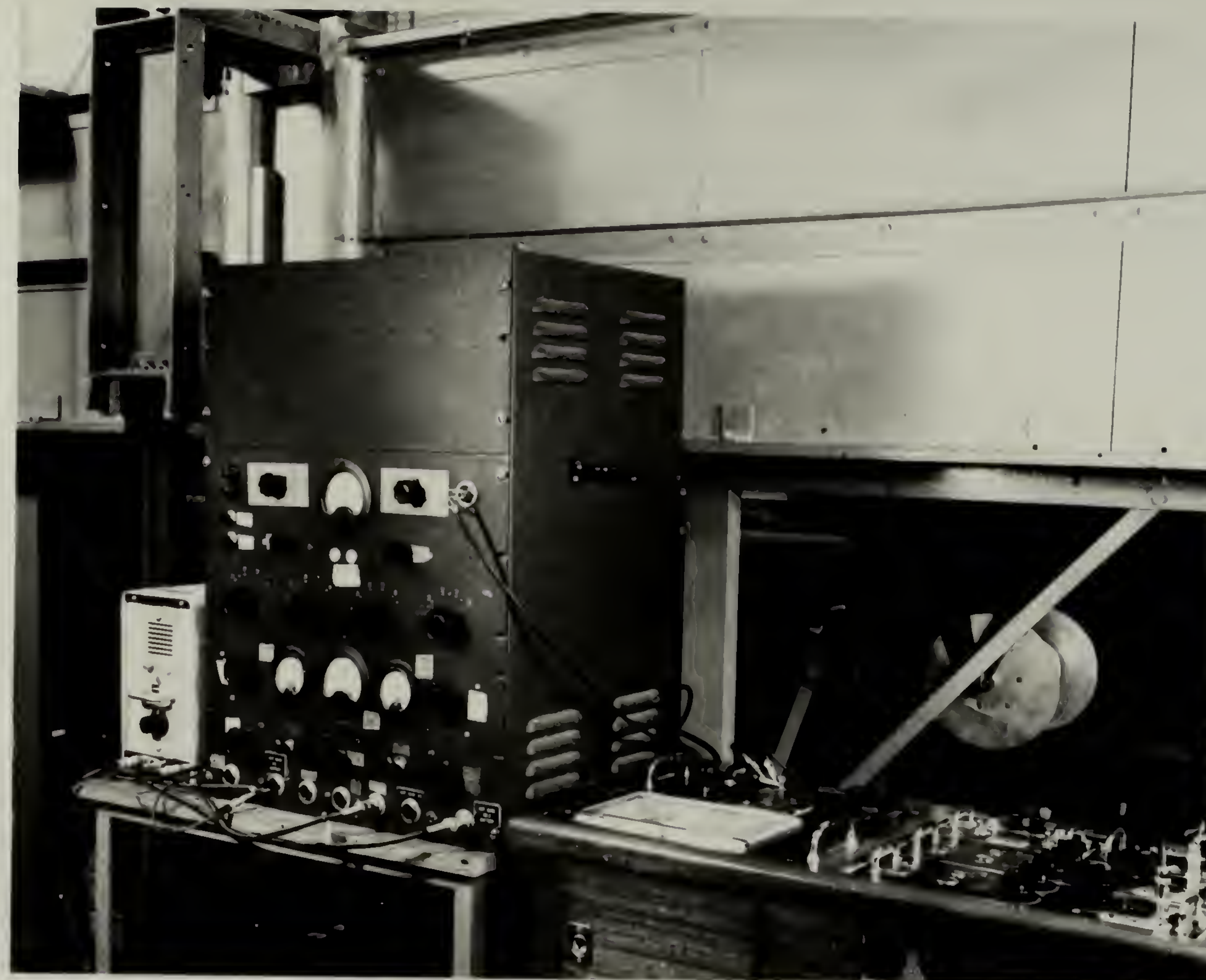
LIST OF ILLUSTRATIONS



Item "A" Item "B" Item "C"

FIGURE 1

General Layout of Apparatus and the Associated Electrical Equipment

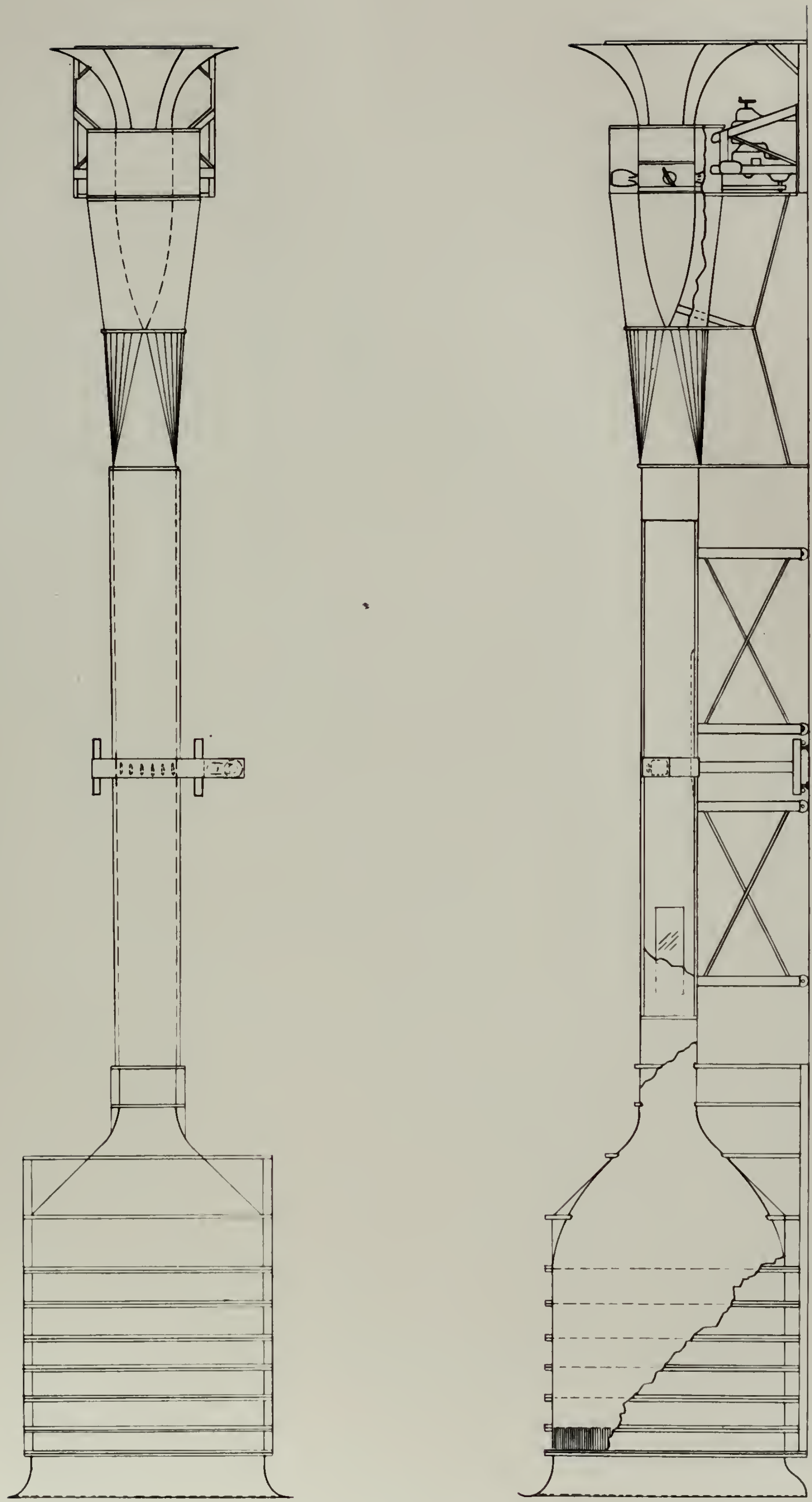


FIG. 2 OPEN RETURN TUNNEL, SINUSOIDAL GUST GENERATOR INSTALLED.

FIGURE 4

View of Linkage Mechanism of Harmonic Gust Generator

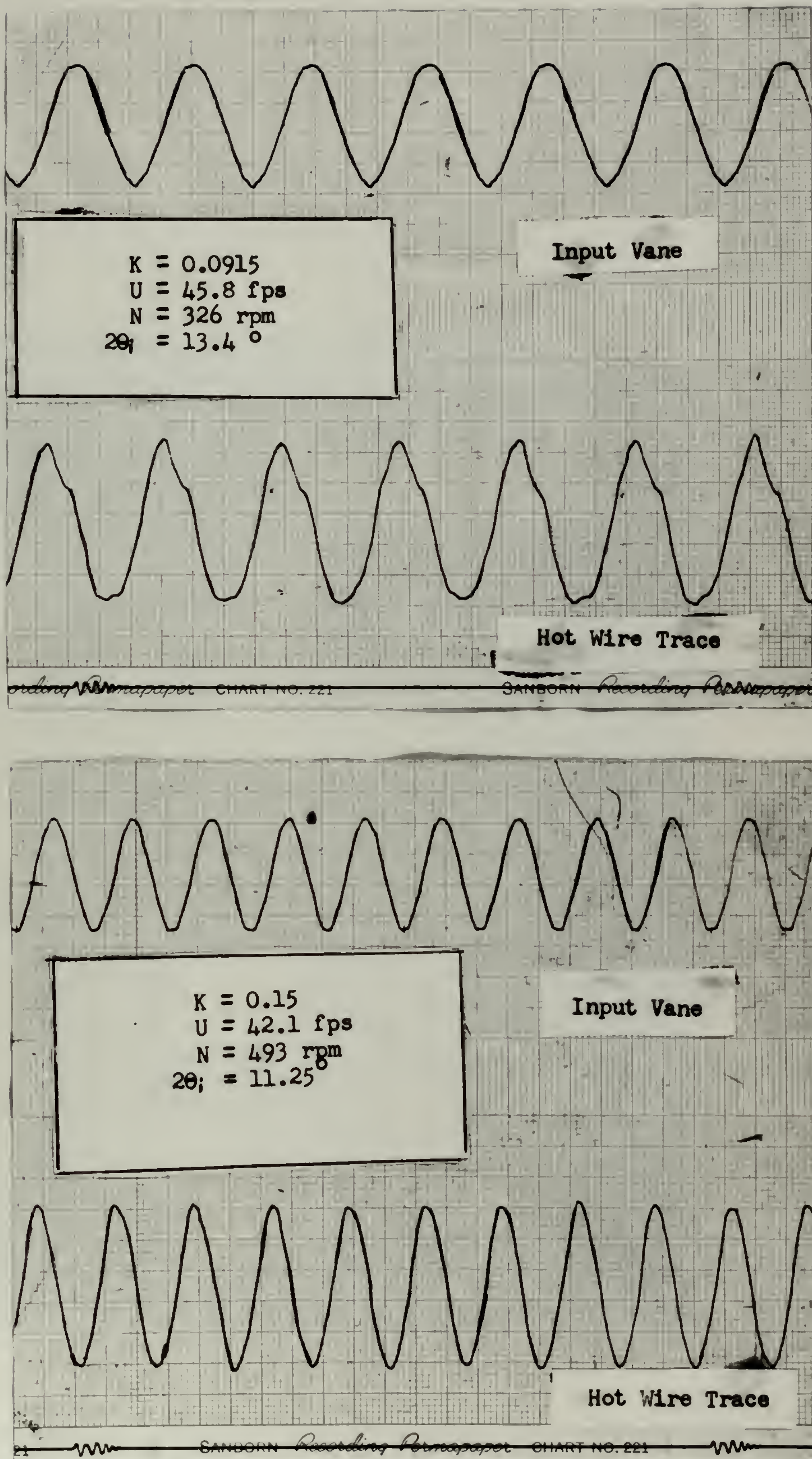


Figure 5 Sample Recordings of Vane Motion and Flow Pattern Produced.

FIGURE 6

View of Hot-Wire Probe and Protractor Scale

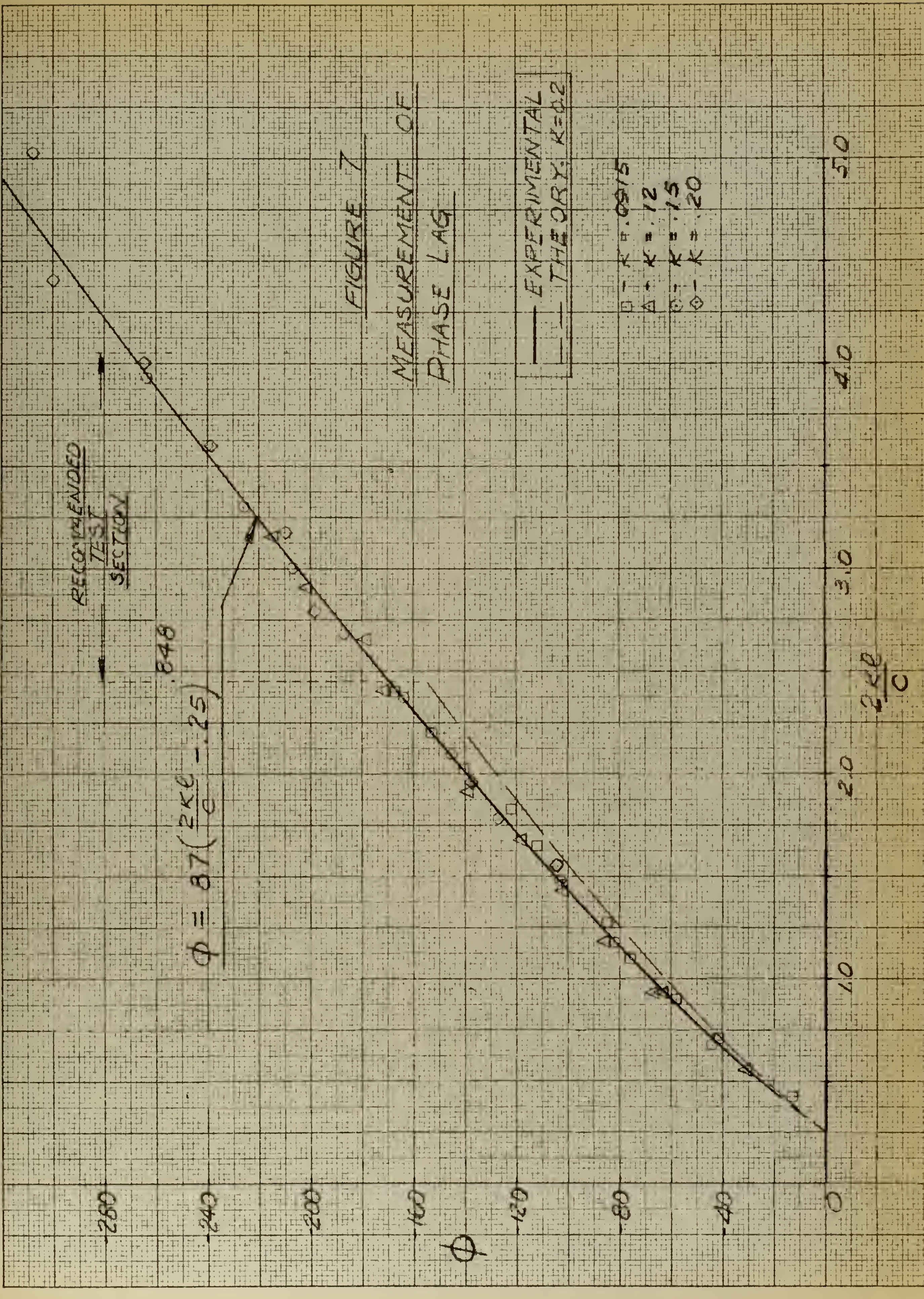


FIGURE 7
MEASUREMENT OF
PHASE LAG
EXPERIMENTAL
THEORY, K=0.2
□ - K = .0975
△ - K = .12
⊙ - K = .15
◇ - K = .20
RECOMMENDED
TEST
SECTION
.848
Φ = 87(2Kℓ/c − .25)
Φ
0
40
80
120
160
200
240
280
1.0
2.0
3.0
4.0
5.0
2Kℓ/c

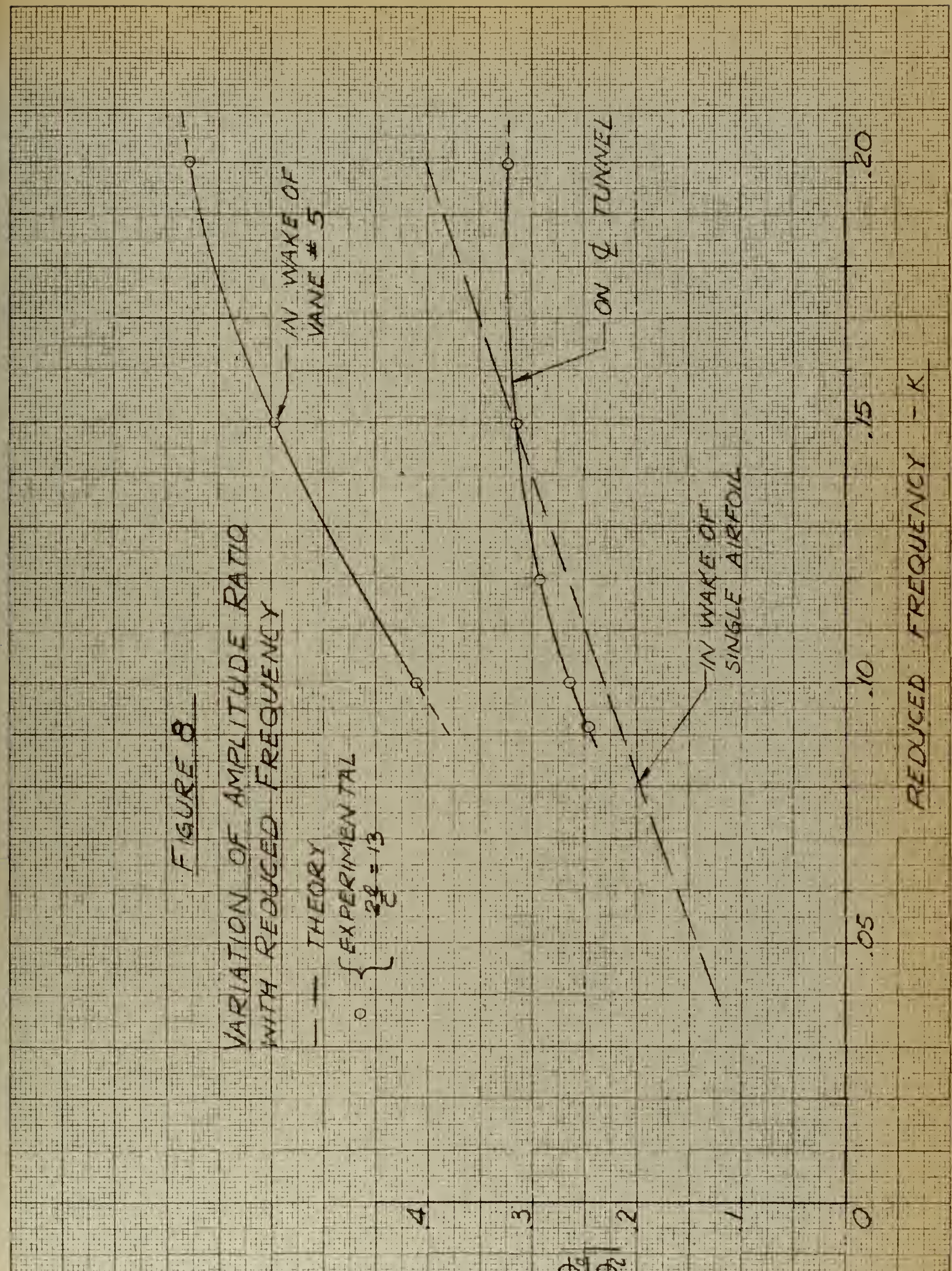
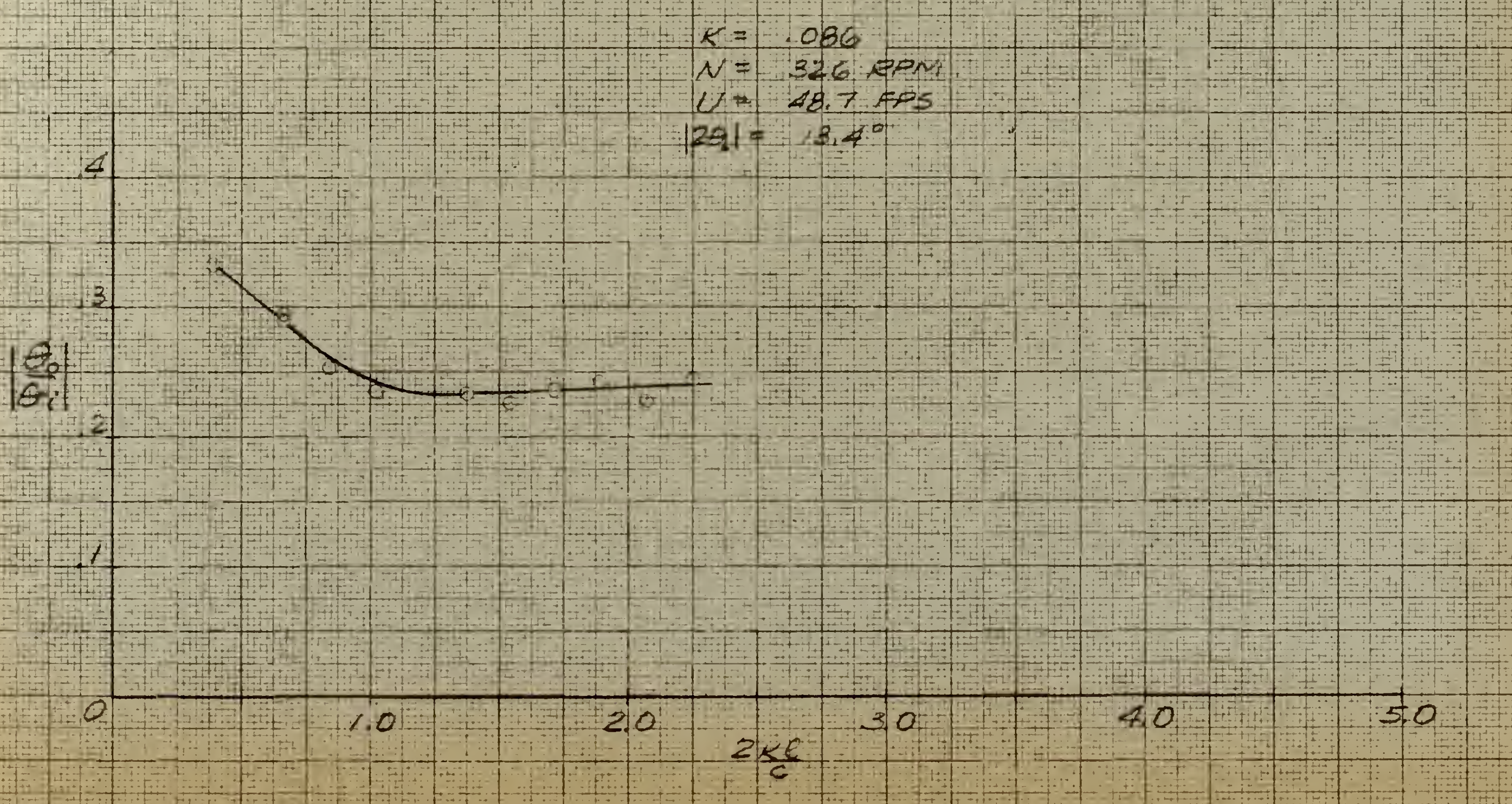


FIGURE 8
VARIATION OF AMPLITUDE RATIO
WITH REDUCED FREQUENCY
— THEORY
○ EXPERIMENTAL
2d/c = 13
IN WAKE OF VANE # 5
ON ℄ TUNNEL
IN WAKE OF SINGLE AIRFOIL
0
1
2
3
4
.05
.10
.15
.20
REDUCED FREQUENCY - K

FIGURE 9

MEASUREMENT OF AMPLITUDE RATIO



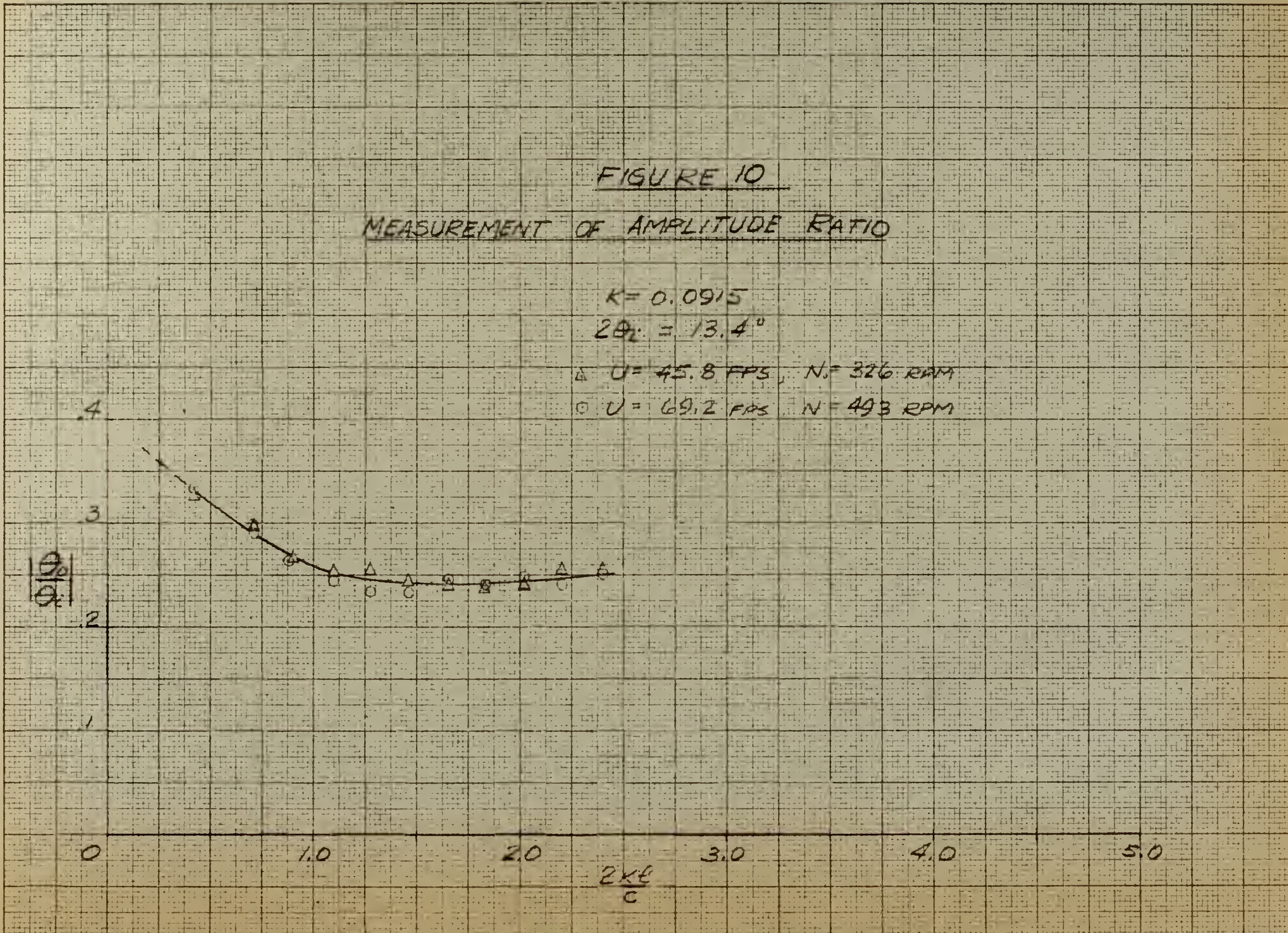
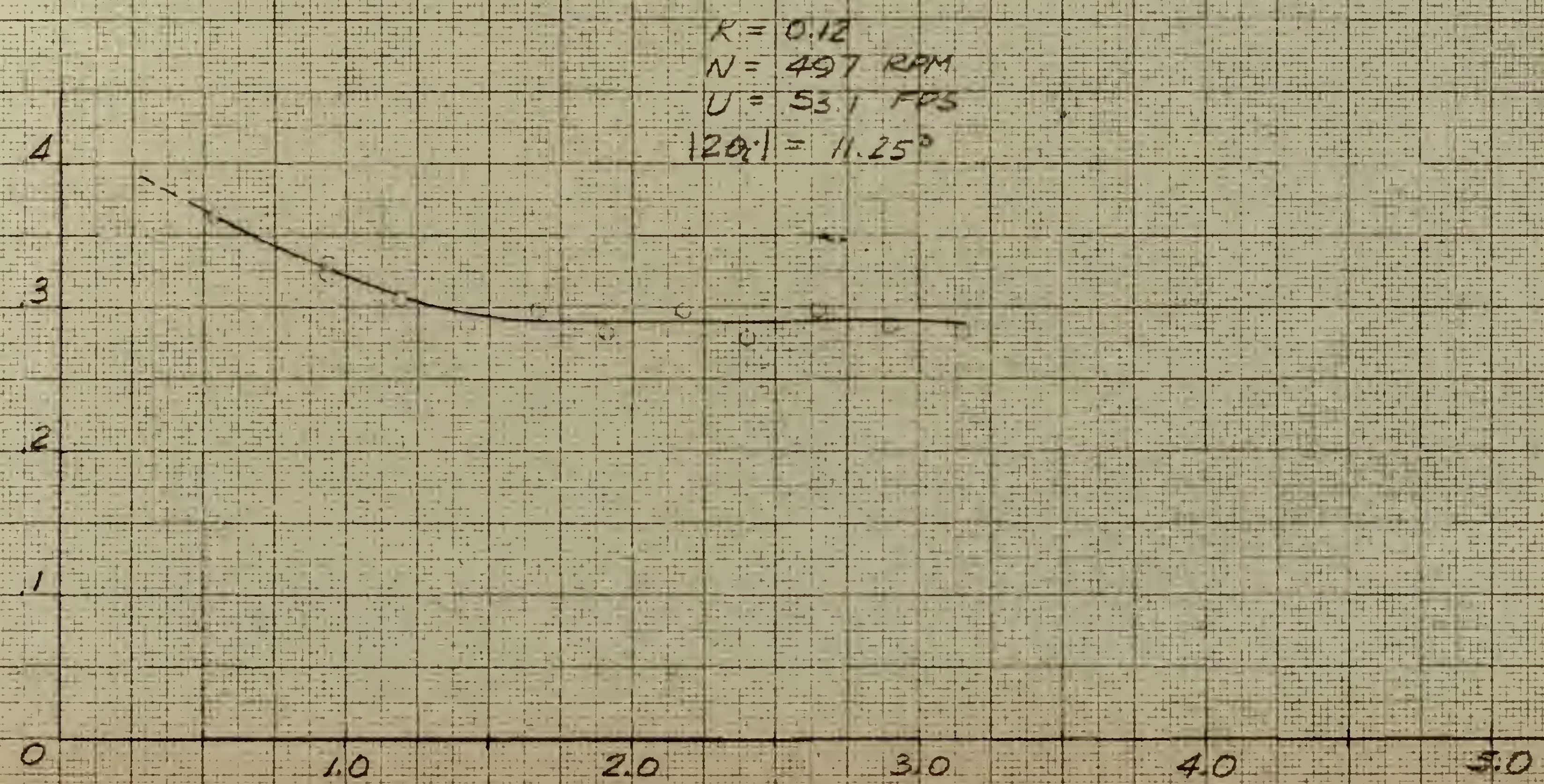


FIGURE 10

MEASUREMENT OF AMPLITUDE RATIO

FIGURE 11

MEASUREMENT OF AMPLITUDE RATIO



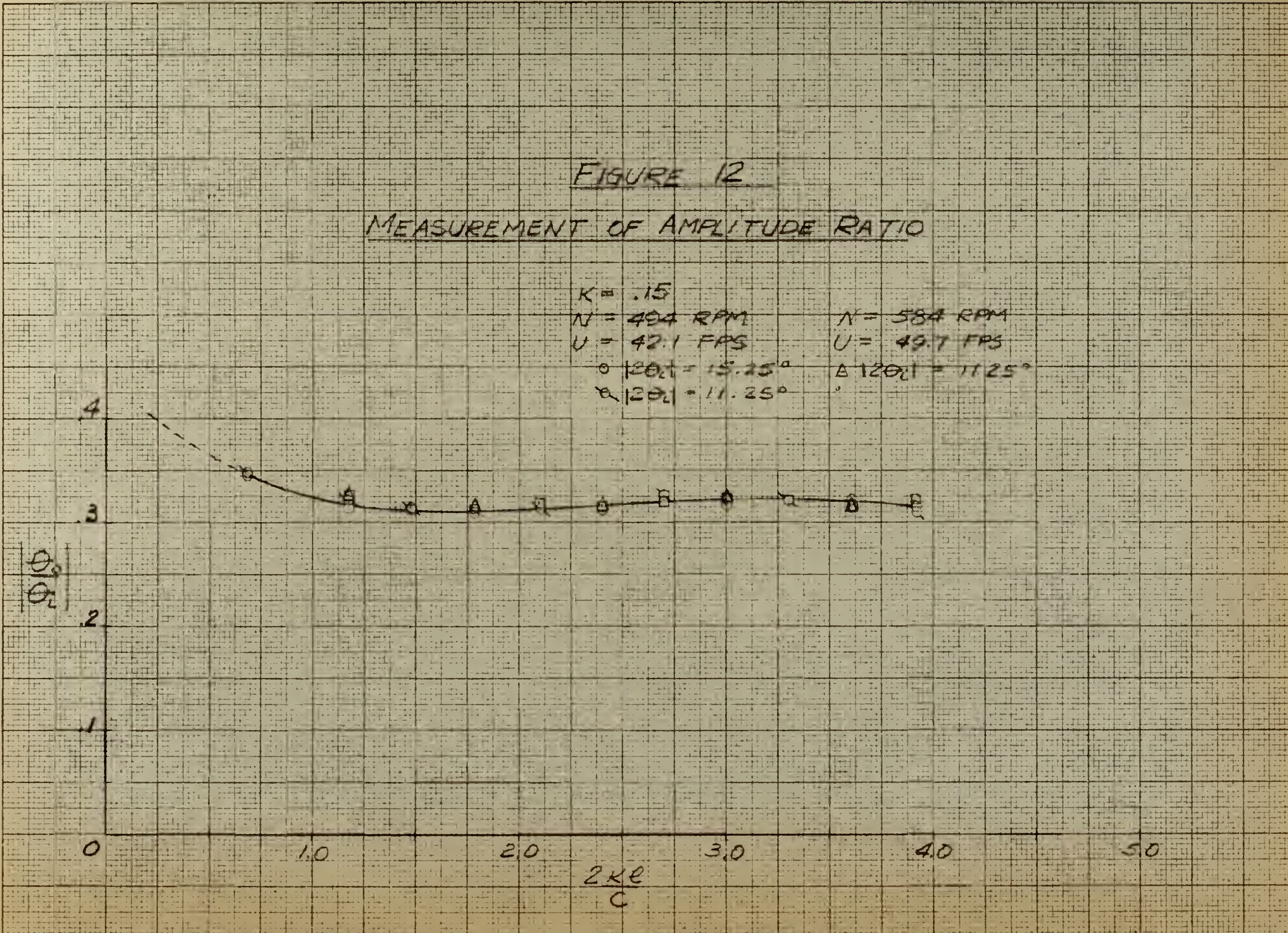


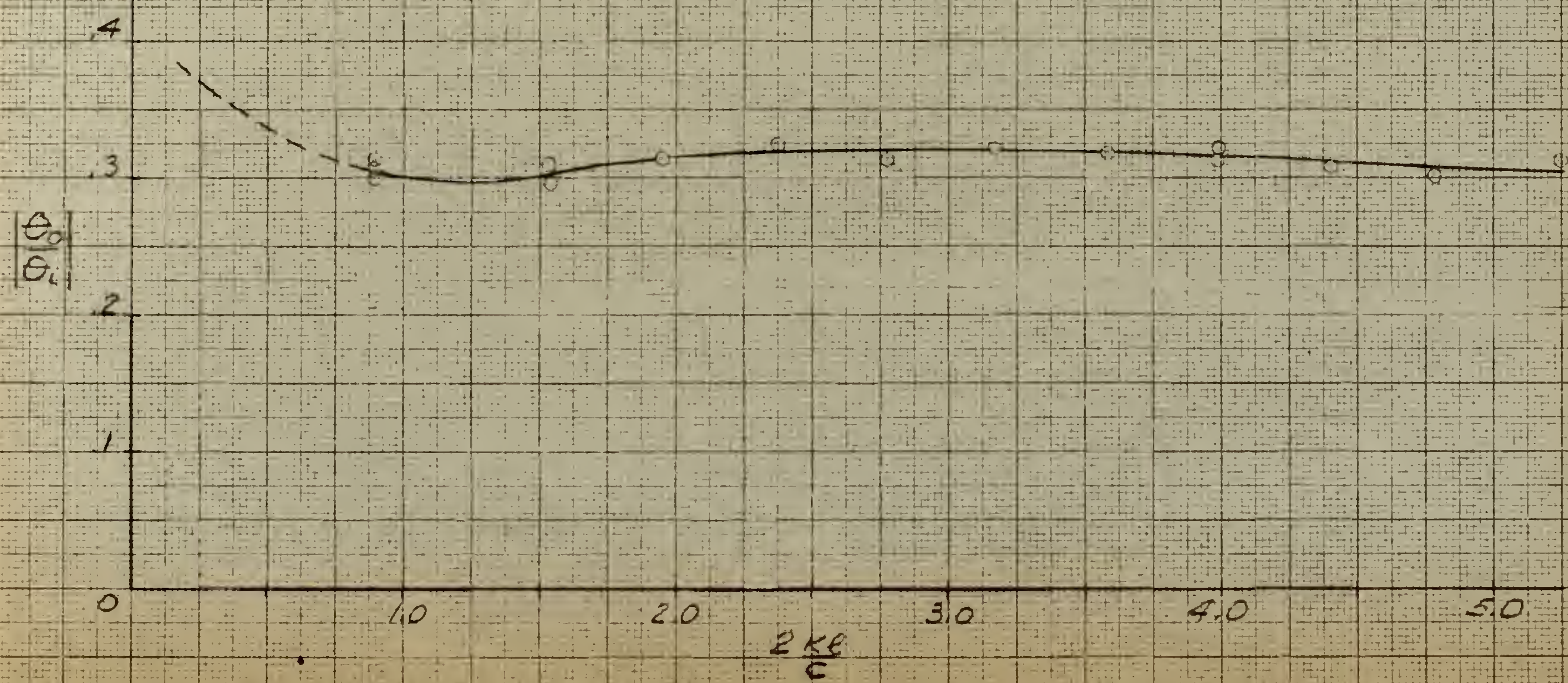
FIGURE 12

MEASUREMENT OF AMPLITUDE RATIO

FIGURE 13

MEASUREMENT OF AMPLITUDE RATIO

K = 0.20
N = 584 RPM
U = 37.3 FPS



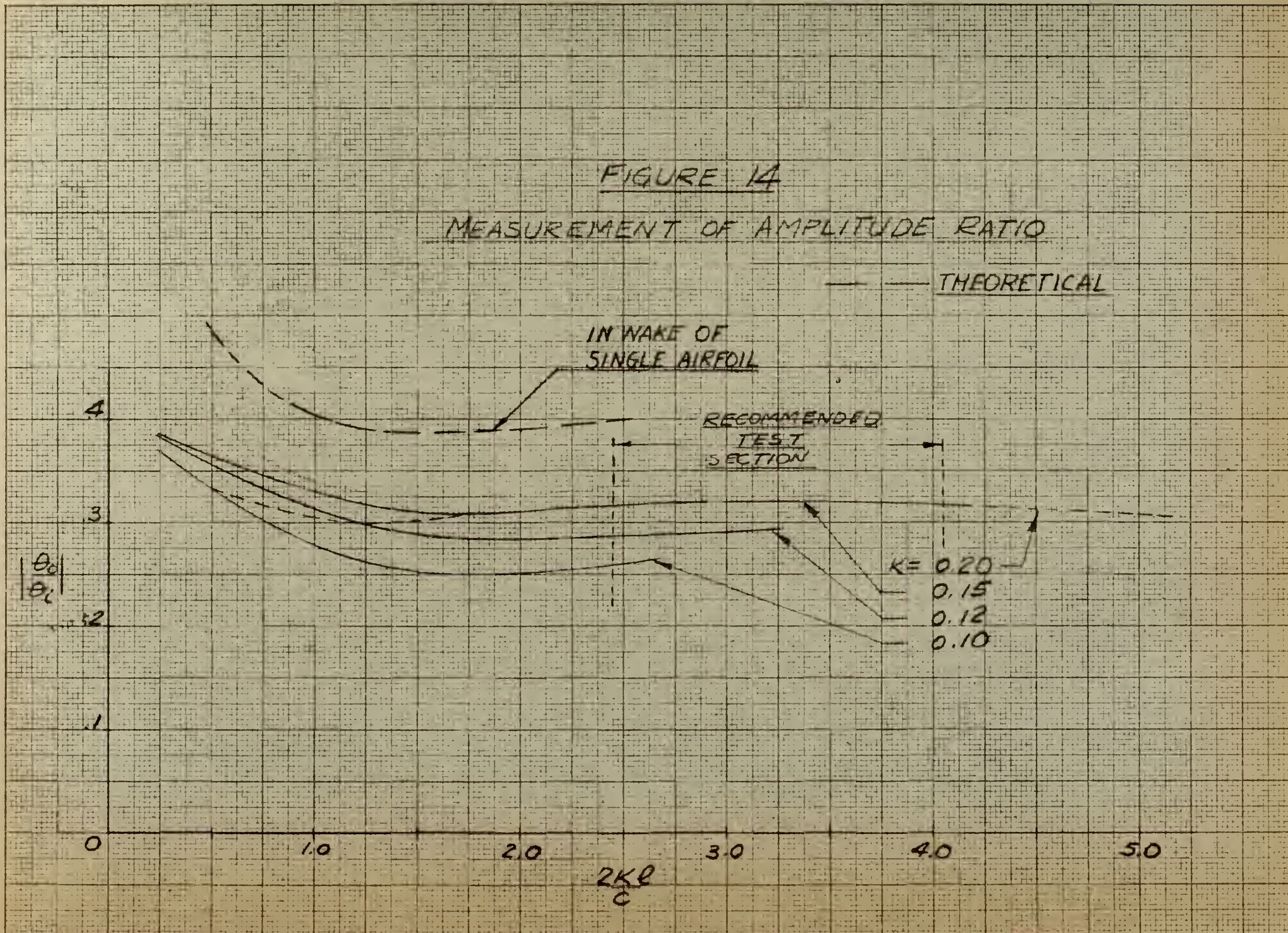


FIGURE 14

MEASUREMENT OF AMPLITUDE RATIO

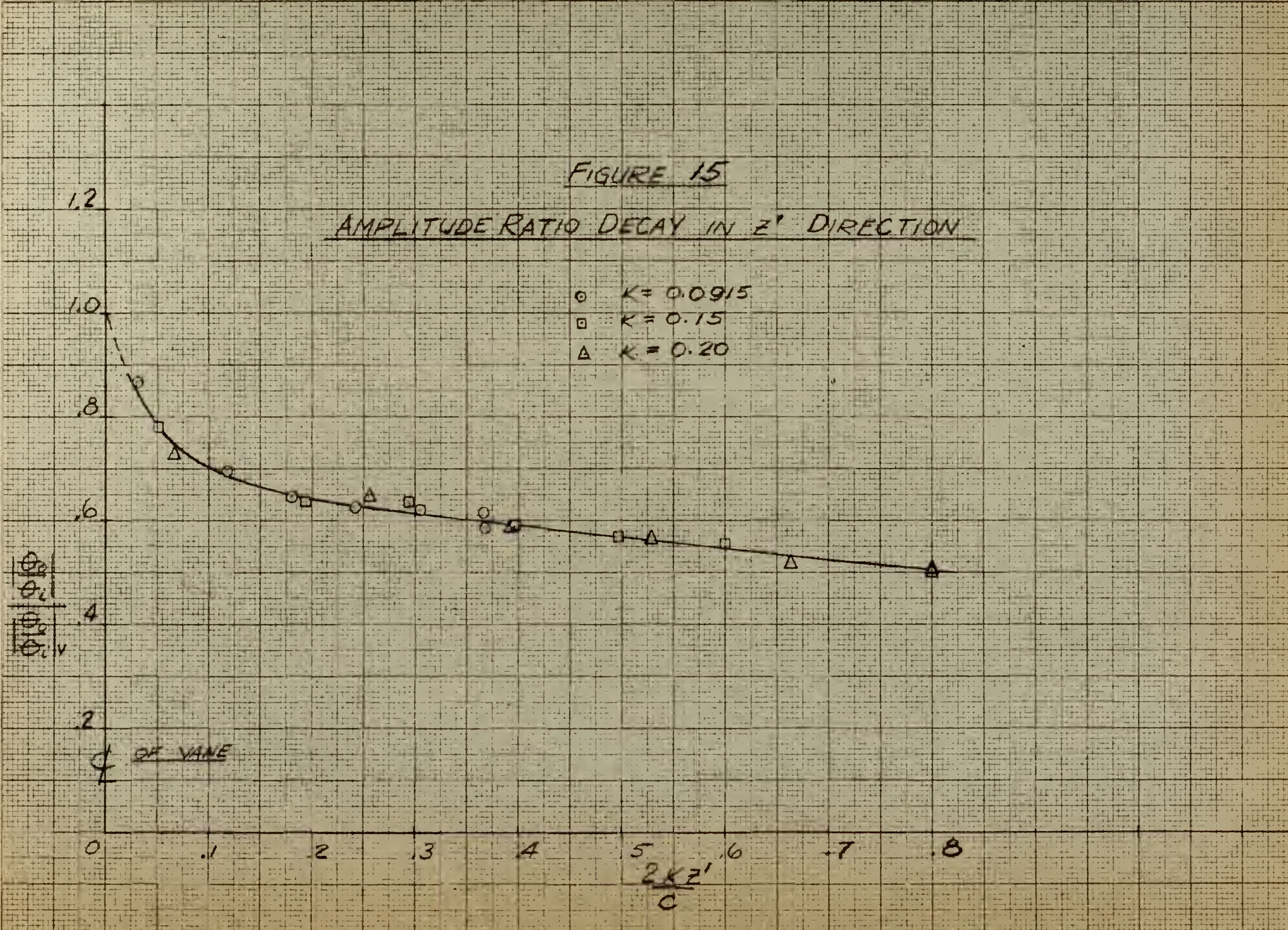
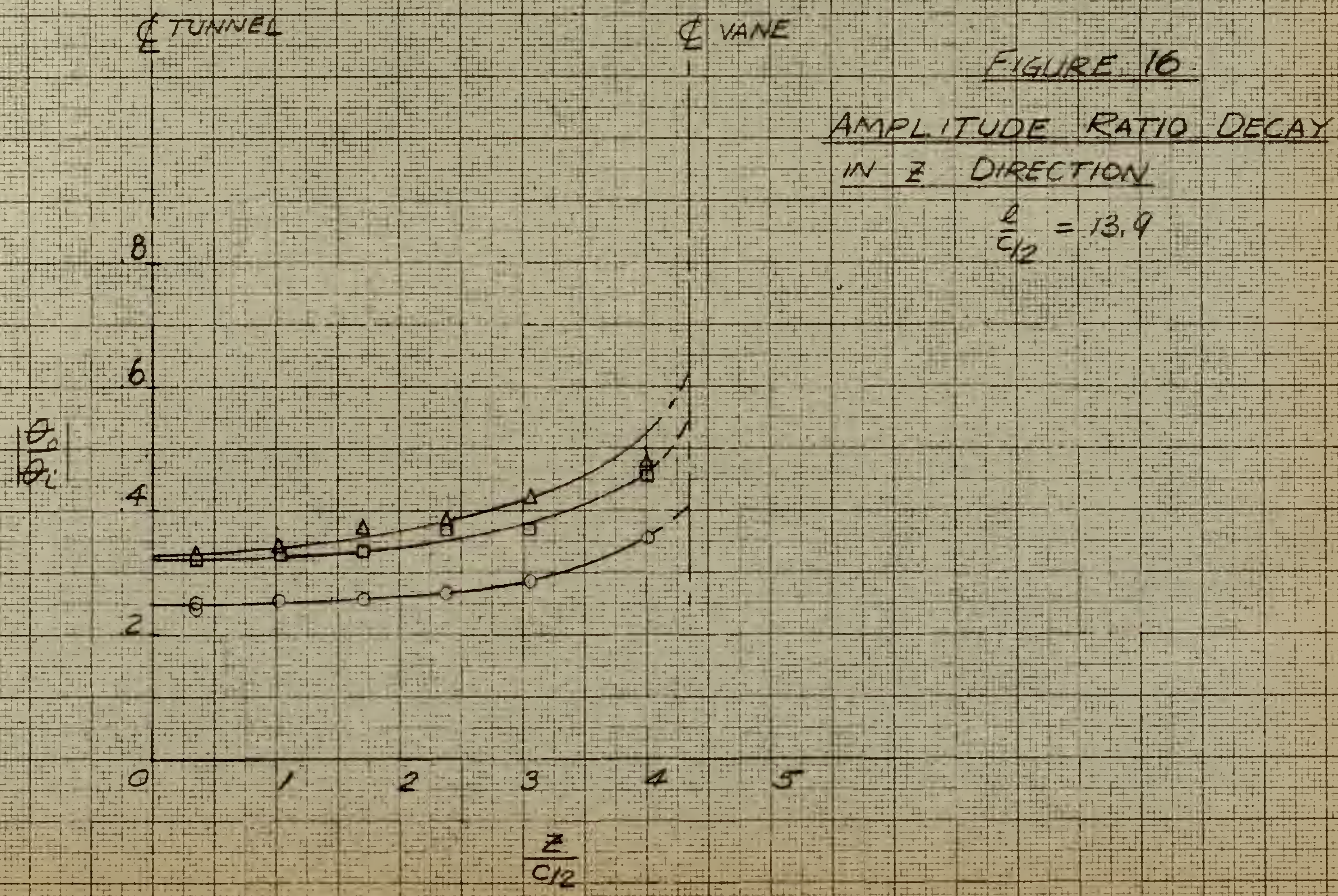


FIGURE 15
AMPLITUDE RATIO DECAY IN Z' DIRECTION
K = 0.0915
K = 0.15
K = 0.20
℄ OF VANE
1.2
1.0
.8
.6
.4
.2
0
.1
.2
.3
.4
.5
.6
.7
.8
2Kz'/C



℄ TUNNEL
℄ VANE
FIGURE 16
AMPLITUDE RATIO DECAY
IN Z DIRECTION
$\frac{\ell}{c/2} = 13.9$
$\frac{\theta_o}{\theta_i}$
8
6
4
2
0
1
2
3
4
5
$\frac{z}{c/2}$

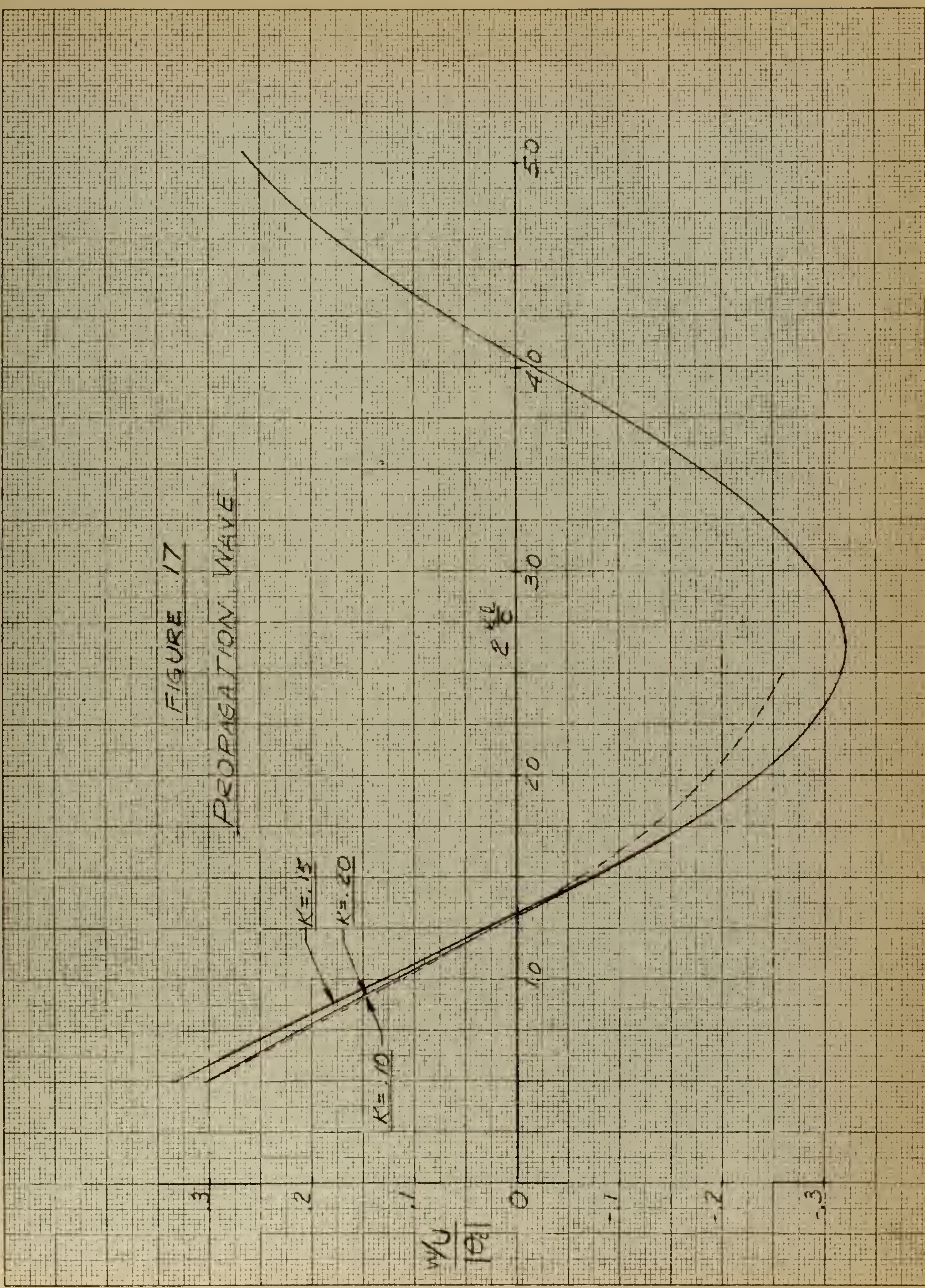


FIGURE 17
PROPAGATION WAVE
K=.10
K=.15
K=.20
10
20
30
40
50
.3
.2
.1
0
-.1
-.2
-.3

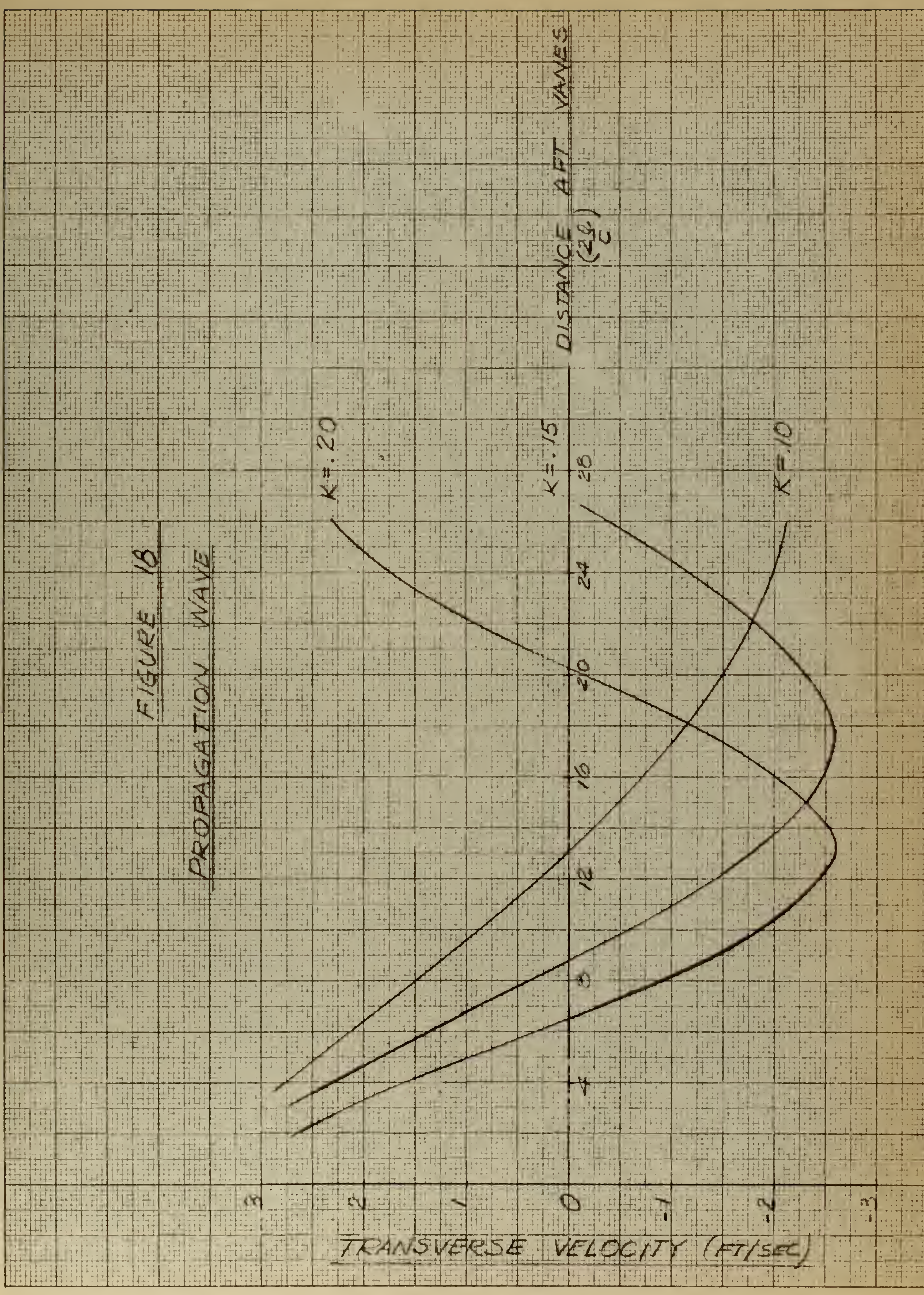
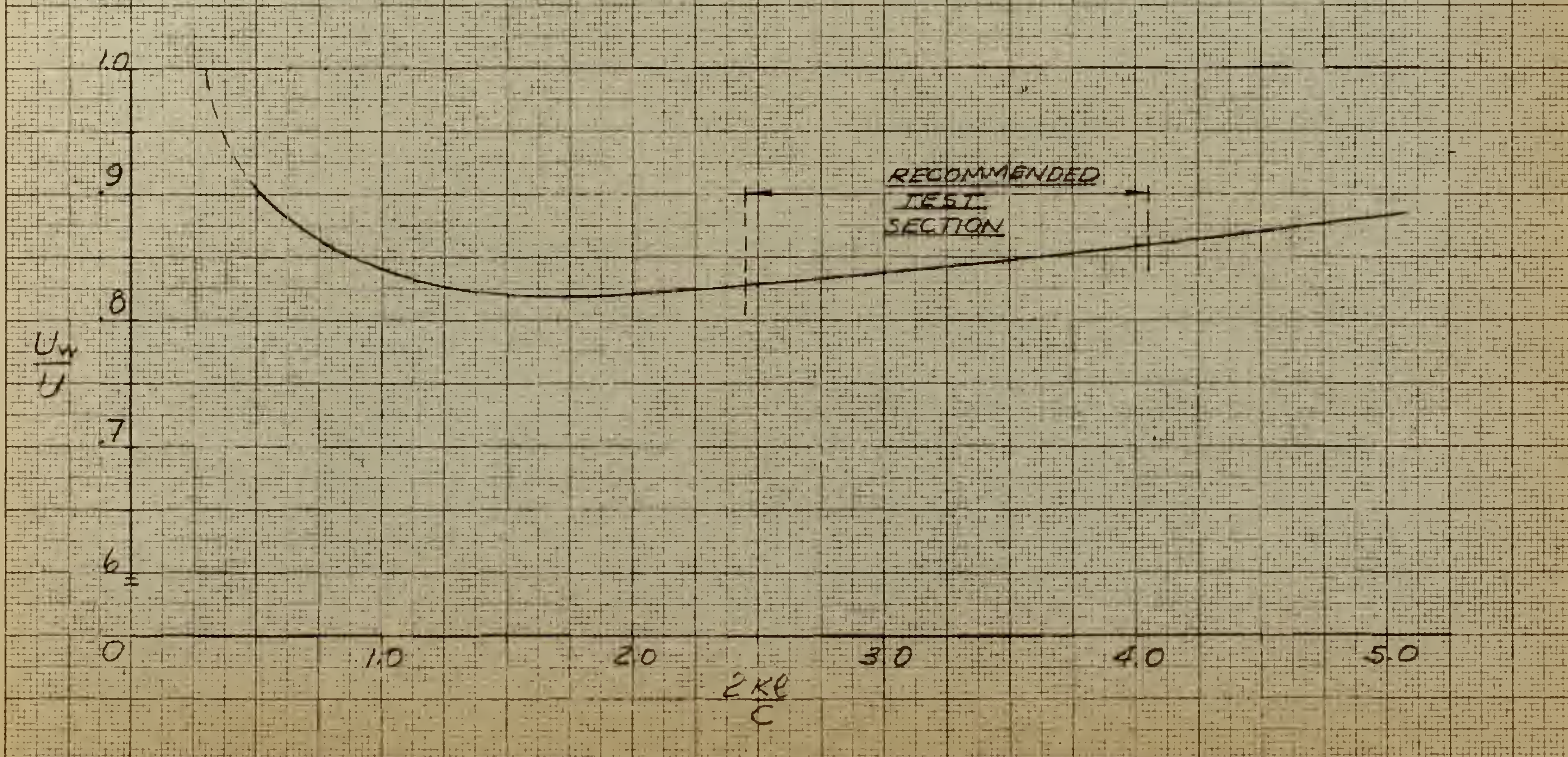


FIGURE 18
PROPAGATION WAVE
DISTANCE AFT VANES $(\frac{2l}{c})$
K=.20
K=.15
K=.10
4
8
12
16
20
24
28
.3
.2
.1
0
-.1
-.2
-.3
TRANSVERSE VELOCITY (FT/SEC)

FIGURE 19

WAVE PROPAGATION VELOCITY



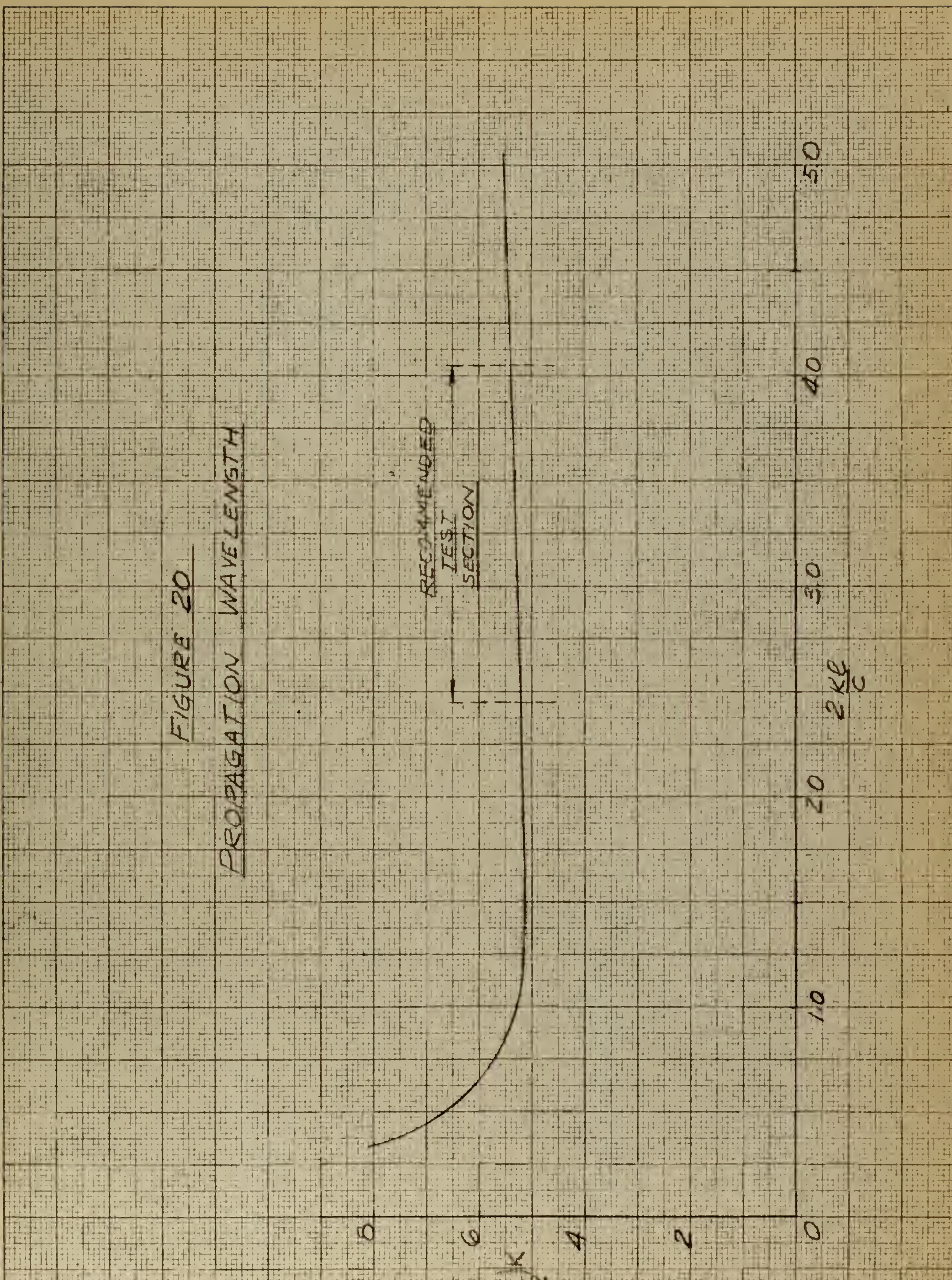


FIGURE 20
PROPAGATION WAVELENGTH
RECOMMENDED
TEST
SECTION
0
10
20
30
40
50
2KE/c
2
4
6
8